

Observability of relative phases of macroscopic quantum states

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Abstract

After a measurement, to observe the relative phases of macroscopically distinguishable states we have to “undo” a quantum measurement. We derive an *inequality* which is satisfied by the relative phases of macroscopically distinguishable states and consequently any desired relative phases can not be observed in interference setups. The principle of *macroscopic complementarity* is invoked that might be at ease with the macroscopic world. We illustrate the idea of limit on phase observability in Stern-Gerlach measurements and the implications are discussed.

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Although the principle of linear superposition is a basic property of microscopic quantum system it has been debated over the years that some macroscopic quantum system can also be found to be in a superposition of all possible states (one example would be Schrödinger's Cat state). A Schrödinger Cat state is a macroscopic object which may be in a *linear superposition of states* corresponding to macroscopically different beings (living and dead) [1]. However, when an observation is made on the Cat state it is found either living or dead but not both. Thus extending the linearity of quantum mechanics to macroscopic domain conflicts with macroscopic realism. But one can always question the validity of the linear superposition principle of quantum mechanics when it is applied to complicated physical systems consisting of large number of atoms and molecules or physical systems of macroscopic dimension. For example Leggett [2] has discussed this question in detail and other issues such as experimental support in favour of macroscopic quantum phenomena. However, we do not address such issues here. Rather, we assume that in nature physical systems do exist in linear superposition of macroscopically distinguishable (MD) states. Some examples of this are superconducting quantum interference devices [3], the possibility of optical production of Cat states [4] and recently discovered Bose-Einstein condensates [5]. Infact, the issue of detection of relative phase [6] between two BE condensate and coherent quantum tunneling between two BE condensates [7] have been discussed in literature. If the macroscopic quantum system is in a superposed state, then the phase relationship between macroscopically distinguishable states must be observed through interference. Hence it is of fundamental importance to discuss the issue of observability of the relative phases of the macroscopically distinguishable states and limit for realising such superposition states.

Any effort to observe interference effect for macroscopic systems would be a difficult task experimentally, because macroscopic systems often interact dissipatively in a irreversible way with the environment [8] and this causes the loss of phase coherence between different branches of macroscopic states. Another argument against observing relative phases is that the measuring apparatus is so large in size that it is impossible to distinguish between the pure state of the total system (system plus apparatus) and the statistical mixture [9]. Peres [10] has argued that observing relative phases of macroscopically distinguishable states requires measurement of certain explicit time-dependent operators and measurement of classical analog of the operator would violate classical irreversibility. It is therefore apparent that all the arguments against observing phases of macroscopic states involve irreversibility in some form or the other and is stated as a sufficient condition for the loss of phase coherence between different branches of the macroscopic states. However, there has not been any attempt to answer the question of (un)observability of the relative phases of macroscopic

systems using fundamental principles of quantum theory.

In this paper we raise an important and fundamental question: Is there an intrinsic quantum limit on the observability of the phases of macroscopically distinguishable states? It turns out that there is a *limit on the observability of the phases of macroscopic states*. The inequality that we derive would put some restriction on the phase relationship between different branches of macroscopically distinguishable states and on the observability of the off-diagonal matrix elements of the pure state density operator. Since the presence of phase coherence distinguishes a pure state from a mixture *the new inequality on phase relationship will be a criterion for the purity of the macroscopic quantum states*. Further, we invoke a *principle of macroscopic complementarity* which would lead to emergence of classical reality for sufficiently large macroscopic systems. We apply these ideas for Stern-Gerlach type measurement and discuss the implication of the inequality for realising the superposition of macroscopically distinguishable states.

In the forgoing discussion unlike Bohr's doctrine the measurement apparatus is not treated classically and quantum mechanical laws are applied to the system as well as apparatus. Suppose we design an apparatus to measure the value of an observable O of a system. The state of the system $|\phi\rangle \in \mathcal{H}_s$ has its own eigenstates $\{|\phi_n\rangle\}$ which forms a complete set and eigenvalues $\{O_n\}$ and the initial state of the apparatus is $|\psi\rangle \in \mathcal{H}_a$. If the initial state of the system is in a linear superposition of the form $|\phi\rangle = \sum_n c_n |\phi_n\rangle$, then the combined state of the system plus apparatus can be written as $\sum_n c_n |\phi_n\rangle \otimes |\psi\rangle$. As a result of interaction between the system and apparatus the final state of the apparatus is different from $|\psi\rangle$ and the combined state is written as $\sum_n c_n |\phi_n\rangle \otimes |\psi_n\rangle$. If the apparatus pointer has to associate a distinct and definite eigenvalue O_n of the observable of the system to some apparatus state $|\psi_n\rangle$, then on physical ground we require the states $\{|\psi_n\rangle\}$ should be mutually orthogonal and macroscopically distinguishable [11]. Thus the macroscopic apparatus during the interaction process has evolved into a linear superposition of macroscopically distinguishable states. Now, the observation of phase relationship between macroscopically distinguishable states would mean observing quantities like $c_i^* c_j (i \neq j)$ which are nothing but the off-diagonal matrix elements of the pure state density operator ρ , i.e. $(\rho_{ij}) = c_i^* c_j$, since interference effects are contained in the off-diagonal elements of the density operator.

Let us consider a general model of measurement where we want to measure an observable O of a system attached to a macroscopic apparatus. The pointer of the apparatus has (center-of-mass) coordinate q , conjugate momentum p and initially localised around $q = 0$. The pointer has to move a macroscopic length L_n to measure the eigenvalue O_n , where $L_n = LO_n$. The total Hamiltonian of the combined system would be $H_T = H_s \otimes 1_a + 1_s \otimes H_a + H_c$, where

H_s and H_a are system and apparatus Hamiltonian, respectively. During the interaction only the coupling Hamiltonian $H_c = V(t)Op$ is important, where $V(t)$ is very large quantity (velocity of the pointer) and $L = \int V(t)dt$. We can write the combined state of the system and apparatus before interaction as $\sum_n c_n |\phi_n\rangle \otimes |\psi\rangle$. In position representation the wave function of the apparatus could be $\langle q, q_2, \dots, q_N | \psi \rangle = \psi(q, q_2, \dots, q_N)$ where q_2, \dots, q_N are regarded as large number of “irrelevant” or “inactive” degrees of freedom [10]. After coupling the state of the combined system is

$$|\Psi\rangle = \sum_n c_n |\phi_n\rangle \otimes e^{-iLpO_n} |\psi\rangle. \quad (1)$$

To measure the eigenvalue O_n we have to look for the pointer position coordinate q . The expectation value of the observable O is still given by

$$\langle \Psi | O | \Psi \rangle = \sum_n |c_n|^2 O_n. \quad (2)$$

The pointer of the apparatus would be at $q = L_n$ with probability $|c_n|^2$. Now we discuss the question of observability of relative phase of different branches of macroscopically distinguishable states after the interaction of the system and apparatus is over. We look for the operator whose expectation value would give the information about the relative phases of different branches.

To derive the limit on the relative phases of different branches we introduce two hermitian operators which is a measure of the phases of the i th and j th branches. Let us define A_1 and A_2 as

$$\begin{aligned} A_1 &= \frac{1}{2} (e^{-iL_i p/\hbar} |\phi_i\rangle \langle \phi_j| e^{iL_j p/\hbar} + e^{-iL_j p/\hbar} |\phi_j\rangle \langle \phi_i| e^{iL_i p/\hbar}) \\ A_2 &= \frac{i}{2} (e^{-iL_j p/\hbar} |\phi_j\rangle \langle \phi_i| e^{iL_i p/\hbar} - e^{-iL_i p/\hbar} |\phi_i\rangle \langle \phi_j| e^{iL_j p/\hbar}), \end{aligned} \quad (3)$$

with $A_1^2 = A_2^2 = \frac{1}{4}(P_i + P_j)$ where P_i and P_j are the projection operators corresponding to eigenstates $|\phi_i\rangle$ and $|\phi_j\rangle$. The expectation value of these operators in the state $|\Psi\rangle$ just after the measurement is given by

$$\begin{aligned} \langle \Psi | A_1 | \Psi \rangle &= \frac{1}{2} (c_i^* c_j + c_i c_j^*) = |c_i| |c_j| \cos \phi_{ij} \\ \langle \Psi | A_2 | \Psi \rangle &= \frac{i}{2} (c_i c_j^* - c_i^* c_j) = |c_i| |c_j| \sin \phi_{ij}, \end{aligned} \quad (4)$$

where ϕ_{ij} are the relative phases of i th and j th branches.

To observe the interference pattern we have to wait for some time after the system has interacted with the apparatus and then superpose its different branches. As a result the combined state is no longer described by $|\Psi\rangle$. At later time the state will evolve under the action of the Hamiltonian $H = H_s \otimes 1_a + 1_s \otimes H_a$. We assume that the observable O of the system commutes with the system Hamiltonian H_s . The evolved state is now given by

$$|\Psi(t)\rangle = e^{-itH/\hbar}|\Psi\rangle = \sum_n c_n |\phi_n\rangle \otimes e^{-itH/\hbar} e^{-iL_n p} |\psi\rangle. \quad (5)$$

The expectation value of the operators A_1 and A_2 in the state $|\Psi(t)\rangle$ are given by

$$\begin{aligned} \langle \Psi(t) | A_1 | \Psi(t) \rangle &= \frac{1}{2} (c_j c_i^* \langle \psi | e^{iL_i p} e^{itH} e^{-iL_i p} e^{iL_j p} e^{-itH} e^{-iL_j p} | \psi \rangle + c.c.) \\ \langle \Psi(t) | A_2 | \Psi(t) \rangle &= \frac{i}{2} (c_i c_j^* \langle \psi | e^{iL_j p} e^{itH} e^{-iL_j p} e^{iL_i p} e^{-itH} e^{-iL_i p} | \psi \rangle - c.c.). \end{aligned} \quad (6)$$

This can be written as

$$\langle A_1 \rangle = |c_i| |c_j| |Z_{ij}(t)| \cos \Phi_{ij}(t) \quad \langle A_2 \rangle = |c_i| |c_j| |Z_{ij}(t)| \sin \Phi_{ij}(t), \quad (7)$$

where $Z_{ij}(t) = \langle \psi | e^{itH(q+L_i)/\hbar} e^{-itH(q+L_j)/\hbar} | \psi \rangle$ and $e^{iL_i p/\hbar} e^{itH/\hbar} e^{-iL_i p/\hbar} = e^{itH(q+L_i)/\hbar}$ etc. Here, $H(q+L_i)$ and $H(q+L_j)$ are nothing but the Hamiltonian H with coordinate q has been shifted by an amount L_i and L_j . The phase $\Phi_{ij}(t)$ is relative phases of the different branches of macroscopically distinguishable states at any time t which contains the phases of $c_i^* c_j$ as well as that of $Z_{ij}(t)$. Note that $Z_{ij}(t)$ contains macroscopic parameters such as length L , mass M of the pointer apparatus.

To what extent the relative phase information we can retrieve is given by the limit that we derive below on the relative phase of two branches of the macroscopically distinguishable states. We will show that it is not possible to observe any arbitrary phase relationship. Macroscopic states only with certain relative phase difference can be observed in interference set ups. We apply the generalised uncertainty relation to two non-commuting hermitian operators A_1 and A_2 . This is given by

$$\Delta A_1^2 \Delta A_2^2 \geq \frac{1}{4} | \langle [A_1, A_2] \rangle |^2. \quad (8)$$

We evaluate the uncertainties in the state $|\Psi(t)\rangle$ of the combined system at an arbitrary time t . They are given by

$$\Delta A_1^2 = \langle A_1^2 \rangle - \langle A_1 \rangle^2 = \frac{1}{4} (|c_i|^2 + |c_j|^2) - |c_i|^2 |c_j|^2 |Z_{ij}(t)|^2 \cos^2 \Phi_{ij}(t)$$

$$\Delta A_2^2 = \langle A_2^2 \rangle - \langle A_2 \rangle^2 = \frac{1}{4}(|c_i|^2 + |c_j|^2) - |c_i|^2 |c_j|^2 |Z_{ij}(t)|^2 \cos^2 \Phi_{ij}(t) \quad (9)$$

and the expectation value of the commutator is given by

$$\langle \Psi(t) | [A_1, A_2] | \Psi(t) \rangle = \frac{i}{2}(|c_j|^2 - |c_i|^2) \quad (10)$$

With the help of (9) and (10) we can simplify the inequality for the relative phases as

$$\sin^2 2\Phi_{ij}(t) \geq \frac{(|c_i|^2 + |c_j|^2)|Z_{ij}(t)|^2 - 1}{|c_i|^2 |c_j|^2 |Z_{ij}(t)|^4} \quad (11)$$

which gives the desired limit on the relative phases of different branches of macroscopically distinguishable states. In actual interferometry one generally measures the relative phase shift as a function of $\sin \Phi_{ij}$ and not $\sin 2\Phi_{ij}$. Therefore, we derive an inequality which expresses this fact. The inequality that we derive below is a stronger one. To further tighten the inequality for relative phases we make use of the so called “triangle inequality”. We know that given three arbitrary non-orthogonal vectors $|\Psi_1\rangle$, $|\Psi_2\rangle$ and $|\Psi_3\rangle$ belonging to the Hilbert space $\mathcal{H} = \mathcal{H}_s \otimes \mathcal{H}_a$ we have the following triangle inequality

$$D(\Psi_1, \Psi_2) + D(\Psi_2, \Psi_3) \geq D(\Psi_1, \Psi_3), \quad (12)$$

where $D(\Psi_\mu, \Psi_\nu)$, $(\mu, \nu = 1, 2, 3)$ is the metric defined from the inner product between the vectors $|\Psi_\mu\rangle$ and $|\Psi_\nu\rangle$. The metric is a measure of distance [12] between the vectors $|\Psi_\mu\rangle$ and $|\Psi_\nu\rangle$ defined on the projective Hilbert space $\mathcal{P} = \mathcal{H}/U(1)$ of combined system. For non-normalisable vectors we define it as [13]

$$D(\Psi_\mu, \Psi_\nu) = \left(1 - \frac{|\langle \Psi_\mu | \Psi_\nu \rangle|^2}{\|\Psi_\mu\|^2 \|\Psi_\nu\|^2}\right). \quad (13)$$

where $\|\Psi_\mu\|^2$ is the norm of the vector $|\Psi_\mu\rangle$ and similarly for $|\Psi_\nu\rangle$. This is the Fubini-Study metric which is invariant under all unitary and anti-unitary transformations acting on the Hilbert space \mathcal{H} . Now define three vectors as follows: $|\Psi_1\rangle = |\Psi(t)\rangle$, $|\Psi_2\rangle = A_1 |\Psi(t)\rangle$ and $|\Psi_3\rangle = A_2 |\Psi(t)\rangle$. The distance functions are given by

$$D(\Psi_1, \Psi_2) = 1 - \frac{4|c_i|^2 |c_j|^2 |Z(t)|^2 \cos^2 \Phi_{ij}}{(|c_i|^2 + |c_j|^2)^2} \quad D(\Psi_2, \Psi_3) = 1 - \frac{(|c_i|^2 - |c_j|^2)^2}{(|c_i|^2 + |c_j|^2)^2},$$

and

$$D(\Psi_1, \Psi_3) = 1 - \frac{4|c_i|^2 |c_j|^2 |Z(t)|^2 \sin^2 \Phi_{ij}}{(|c_i|^2 + |c_j|^2)^2}. \quad (14)$$

Inserting (14) in (12) and simplifying the inequality we have

$$\cos 2\Phi_{ij} \leq \frac{1}{|Z(t)|^2(|c_i|^2 + |c_j|^2)} \quad (15)$$

Further, on combining (15) with (11) we have the tightened version of the inequality for the relative phases of the two branches as is given by

$$(\sin^2 \Phi_{ij}) \geq \frac{1}{2} \left(\frac{|Z_{ij}(t)|^2(|c_i|^2 + |c_j|^2) - 1}{|Z_{ij}(t)|^2(|c_i|^2 + |c_j|^2) + 1} \right) \frac{(|c_i|^2 + |c_j|^2)}{|c_i|^2 |c_j|^2 |Z_{ij}(t)|^2} \quad (16)$$

The above inequality is more stronger than (11) as it makes use of uncertainty relation and triangle inequality. To know the bound on the phases we really do not have to measure any operator for that matter. The knowledge of probability distributions (i.e. $|c_i|^2$ and $|c_j|^2$) and expectation value of displaced unitary operators (it contains the macroscopic parameters such as length, mass and possibly other variables of the pointer) is enough to tell us the phase information. This inequality is valid for all times even much after the measurement of the observable O of the system. It restricts the relative phase of macroscopically distinguishable states and *those macroscopic states with a relative phase lower than the predicted value can not be superposed to produce an interference pattern*. Because then that would reveal the observable phase information in contradiction with the uncertainty (inequality) principle. This is a *necessary* criterion for any macroscopically distinguishable states for producing an interference pattern. If the relative phases of different branches violate the above inequality then the quantum interference effect is not important for all practical purposes.

We can understand the limitation if we renunciate the extended Bohr's complementarity for macroscopic systems as follows: *The non-violation of the inequality by relative phases and acquisition of which-state information (assigning a definite macrostate) are mutually exclusive*. This we call as new macro-complimentarity. Jammer [14] has analysed Bohr's view on macro-complimentarity and concluded that the issue between realism and idealism is a matter subject to complementarity. Leggett [15] has discussed the possibility of exhibiting no interference between macroscopically distinct states and in assigning a definite state to a macroscopic system. The principle of macroscopic complementarity consolidates the spirit of earlier works in these lines.

Does the macro-complimentarity shed some light on the macroscopic systems which are under every day-level observation. For example why do we see a real Cat in one or the another state and not in a superposition of different possible states. It seems that for sufficiently large bodies *the relative phases of distinct branches are adjusted so as to violate the strong inequality (16) thereby not allowing to observe the interference between them*. Also, it can be verified that if we can assign a definite state to a macrosystem the inequalities (11) and (16) are violated. Thus, the new inequality together with macro-complimentarity helps us

to understand the macroscopic world as we live. It might be true that a real Cat is in a linear superposition of all of its possible states (living and dead) but the relative phase of the two branches would not satisfy either the uncertainty inequality or the strong inequality. Thus, the principle of macroscopic complementarity helps us to understand the emergence of realism from the quantum mechanical principles.

If we observe the relative phase after a short time t of the interaction then we may assume the relative phases between different branches may be small and one can write $\sin \Phi_{ij}(t)$ as $\Phi_{ij}(t)$. Then from the above inequality one can infer a minimum relative phase as is given by

$$(\Phi_{ij}(t)^2)_{min} = \frac{1}{2} \left(\frac{|Z_{ij}(t)|^2(|c_i|^2 + |c_j|^2) - 1}{|Z_{ij}(t)|^2(|c_i|^2 + |c_j|^2) + 1} \right) \frac{(|c_i|^2 + |c_j|^2)}{|c_i|^2|c_j|^2|Z_{ij}(t)|^2} \quad (17)$$

Is there any limit on the relative phases just after the interaction with the apparatus. From (16) we can see that immediately after interaction the state of the combined system is $|\Psi\rangle$. Hence, the uncertainty inequality has to be evaluated in the state $|\Psi\rangle$ just after the interaction. The inequality is given by

$$\sin^2 \phi_{ij} \geq \frac{1}{2} \left(\frac{(|c_i|^2 + |c_j|^2) - 1}{(|c_i|^2 + |c_j|^2) + 1} \right) \frac{(|c_i|^2 + |c_j|^2)}{|c_i|^2|c_j|^2|Z_{ij}(t)|^2} \quad (18)$$

The significance of the above inequality is that even immediately after preparing a superposition of macroscopically different states we cannot obtain the phase information in any desired way. There is an intrinsic quantum limitation in doing that.

We illustrate the idea of limitation on the phase with the help of a better known example- the Stern-Gerlach like measurement. This example has been considered by Peres [10] in trying to understand the connection between the quantum mechanical and classical irreversibility. Consider a macroscopic apparatus which is designed to measure the z -component of the spin- $\frac{1}{2}$ particle (say an electron). The pointer of the apparatus has (center-of-mass) coordinate q , conjugate momentum p and initially localised around $q = 0$. The motion of the pointer to right or left will decide whether the spin is $\frac{1}{2}$ or $-\frac{1}{2}$. The interaction Hamiltonian is $H_c = V(t)\sigma_z p = 2V(t)s_z p$ where $s_z = \frac{1}{2}\sigma_z$, σ_z being the Pauli spin matrix, $V(t)$ is velocity of the pointer and $L = \int V(t)dt$ (a macroscopic distance). We can write the initial state of the spin-half particle as $|\phi\rangle = (\alpha|+\rangle + \beta|-\rangle)$ and of apparatus as $|\psi\rangle$. After coupling the state of the combined system is

$$|\Psi\rangle = \alpha|+\rangle \otimes e^{-iLp/\hbar}|\psi\rangle + \beta|-\rangle \otimes e^{iLp/\hbar}|\psi\rangle \quad (19)$$

To measure the z -component of the spin we have to look for the sign of the pointer position coordinate q . The expectation value of the s_z is given by

$$\langle s_z \rangle = \frac{1}{2}(|\alpha|^2 - |\beta|^2) \quad (20)$$

The pointer of the apparatus would be at $q = L$ with probability $|\alpha|^2$ and at $q = -L$ with probability $|\beta|^2$. Once a measurement is done we know only of $|\alpha|^2$ and $|\beta|^2$. Here, we discuss the limit on the relative phases of the two branches from a fundamental uncertainty principle.

As discussed earlier at later time the state will evolve under the action of the Hamiltonian $H = H_e + H_a$, where H_e is the electron Hamiltonian and H_a is the apparatus Hamiltonian. The evolved state is now given by

$$|\Psi(t)\rangle = e^{-itH/\hbar}|\Psi\rangle = \alpha|+\rangle \otimes e^{-itH/\hbar}e^{-iLp/\hbar}|\psi\rangle + \beta|-\rangle \otimes e^{-itH/\hbar}e^{iLp/\hbar}|\psi\rangle \quad (21)$$

The operators A_1 and A_2 takes the simple form

$$A_1 = s_x \cos 2Lp + s_y \sin 2Lp \quad A_2 = s_x \sin 2Lp - s_y \cos 2Lp \quad (22)$$

We evaluate the uncertainties in the state $|\Psi(t)\rangle$ of the combined system at an arbitrary time t . They are given by

$$\begin{aligned} \Delta A_1^2 &= \langle A_1^2 \rangle - \langle A_1 \rangle^2 = \frac{1}{4} - |\alpha|^2|\beta|^2|Z(t)|^2 \cos^2 \Phi \\ \Delta A_2^2 &= \langle A_2^2 \rangle - \langle A_2 \rangle^2 = \frac{1}{4} - |\alpha|^2|\beta|^2|Z(t)|^2 \sin^2 \Phi. \end{aligned} \quad (23)$$

where $|Z(t)|$ is given by $\langle e^{itH(q-L)}e^{-itH(q+L)} \rangle = |Z(t)|e^{i\theta(t)}$ and $\Phi(t) = \phi + \theta(t)$, ϕ being the relative phase of α and β . Therefore the uncertainty inequality can be expressed as

$$\sin^2 2\Phi \geq \frac{|Z(t)|^2 - 1}{|\alpha|^2|\beta|^2|Z(t)|^4} \quad (24)$$

We can express the tightened version of the inequality (16) for the relative phases of the two branches in Stern-Gerlach measurement as

$$(\sin^2 \Phi) \geq \frac{1}{2} \left(\frac{|Z(t)|^2 - 1}{|Z(t)|^2 + 1} \right) \frac{1}{|\alpha|^2|\beta|^2|Z(t)|^2} \quad (25)$$

which gives the desired lower bound on the relative phases of two macroscopically distinguishable states.

Some consequence of the above inequality relation can be discussed now. If we ask what is the detectable phase just after the ‘‘premeasurement’’. In that case the inequality has to be evaluated with the state (18) and $Z(t)$ is just equal to one. The inequality says that

$\sin^2 \phi \geq 0$ which is trivial. The same would be true if the apparatus Hamiltonian does not depend on q . Therefore for a non-trivial lower bound on the relative phase we require the state to evolve for an appreciable period of time and it is necessary that the Hamiltonian H depends on q . The dependence of H on q means the pointer of the apparatus is moving under some potential $V_a(q)$ and the energy thus varies from place to place as time progresses. Interestingly, one can check that if we assign a definite state to the macroscopic system (say $|\alpha|^2 = 1$ and $|\beta|^2 = 0$) the inequality (25) is violated. This is in agreement with macroscopic complementarity, stated earlier.

Here, we briefly discuss the argument of Peres for undoing a quantum measurement and show that it is not a serious objection against observing relative phases. His argument runs as follows: Let us define an operator $A = A_1 + iA_2$ and the expectation value of the operator A in this state is given by

$$\langle A \rangle = \alpha\beta^* \int \psi^* e^{-iLp/\hbar} e^{itH/\hbar} e^{2iLp/\hbar} e^{-itH/\hbar} e^{-iLp/\hbar} \psi d^N q \quad (26)$$

This can be written as

$$\langle A \rangle = \alpha\beta^* \langle e^{itH(q-L)/\hbar} e^{-itH(q+L)/\hbar} \rangle \quad (27)$$

It can be shown that for short time (for a proper choice of time), $\langle A \rangle$ can go to zero and the relative phases of the two macroscopic states is lost after some finite time. However, one can measure another operator A' , where

$$A' = e^{-itH/\hbar} A e^{itH/\hbar} \quad (28)$$

and its expectation value is nothing but $\alpha\beta^*$. But such an operator is explicitly time dependent constant of motion. Classically (for a system with N degrees of freedom we have $2N$ constants of motion)) such constants of motions are large compared to constants of motion which are explicitly time-independent. Such constants of motion are of no use to us because they are quite complicated for large N and finite time. This results in unpredictability of the initial position and momentum and hence in irreversibility. Therefore, Peres concludes that the measurement of classical analog of operator (which gives the relative phases) would mean the violation of classical irreversibility.

But I believe that such argument against “undoing” a quantum measurement is not a serious one. First of all not every quantum mechanical operator has a classical analog although the converse is true. The best example is the spin of a particle which has no classical analog. Indeed the operator one would measure to reveal the relative phase is related to the components of the spin operator and it is not expected to have classical analog. Therefore

any violation in classical world would not prohibit “undoing” a quantum measurement. Further, it has been proved by Wigner [16] that an operator which does not commute with a conserved quantity can not be measured exactly (in the sense of von Neumann). In the discussion of Peres *neither the operator A nor A' commute with a conserved quantity s_z* . The z-component of the electron is a conserved quantity assuming that H_e does not contain any spin component other than s_z . Therefore, even in principle the operator A or A' can not be measured exactly. As a result the exact relative phase can not be obtained by measuring the explicit time-dependent operator A' . Hence, one should look for an estimate of the relative phase of the two branches, which is precisely what we have aimed at.

Thus, in conclusion we have discussed the limitations for realising macroscopic quantum superpositions. We have derived an *inequality* concerning the observability of the relative phase of macroscopically distinguishable states. *If linear superposition principle holds for macroscopic states then the inequality has to be necessarily satisfied* by the relative phases. It is suggested that the new inequality can be taken as a criterion for the purity of a macroscopic quantum state. We invoked the idea of macro-complementarity which may help to understand how does a macroscopic system come into a definite state and it may resolve the issue of unobservability of interference between different possible (real) Cat states. We argued that violation of classical irreversibility is not always (at least in the example considered) serious objection against “undoing” a quantum measurement.

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